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Exponential Observer for Nonlinear System Satisfying Multiplier Matrix

Wei Guo[1], Xiang Wu[2]

[1] (*College Of Mechanical Engineering, Shanghai University Of Engineering Science, Shanghai 201620, China*)
[2] (*College Of Mechanical Engineering, Shanghai University Of Engineering Science, Shanghai 201620, China*)

Abstract: In this article, we design an observer for a class of systems with multivariable nonlinearities satisfy incremental multiplier matrix and scalar nondecreasing characteristic .We will make full use of the nonlinear part of the structure, and relax the limit of the linear part of the observer error system.Nonlinear part is parameterized by a set of multiplier matrices. Observer design is simplified to solve linear matrices.The proposed observer error converges to zero exponentially; Conclusions will be liste.

Keywords: nonlinear , multiplier matrix,state estimation,models,observer

I. Introduction

One of the basic problems in system analysis and control design is determining the state of the system from its measurement input output. Many solutions to this problem use a progressive observer estimate of the system asymptotically close to the state of the actual system. At the same time , the solutions of nonlinear systems should be convergence and boundedness .They are important issues in system analysis and control design. The observer of the linear system is composed of a copy of the system which along with a linear correction section which based on output error. Luenberger [1]proposed this type of observer.In order to make the observer asymptotically stable, the error can be realized by controlling the state estimation error.To obtain linear observer error dynamics, Krener and Isidori [2] initiateda geometric design, which has been further studied by numerous authors, including Kazantzis and Kravaris [3],whose procedure was less-conservative.The rest of the study includes open-loop observers, explored by Lohmiller and Slotine [4]; and an H∞-like design for nonlinearities with linear growth bounds, introduced by Thau.Recently proposed observer design by Arcak and Kokotovic [5] has been removed a long term existence linear growth assumption fromnonlinearities of the unmeasurable states.He has put forward

That Convergence of the estimates to the real states is implemented under two qualification which allow the observer error system to meet thecircle criterion:In the first place, a linear matrix inequality (LMI) should be viable,it can guarantee a strict positive real property for the linear part of the observer error system.Second qualified conditions is that the nonlinearities should be monotone increasing function of the unmeasured states.

Arcak and Kokotovic [6],Hammouri,Gauthier, and Othman[7], Hedrick and Raghavan[8], and Rajamani [9] develop asymptotic observer synthesis methods for which nonlinear systems with globally Lipschitznonlinearities and nonlinearities in unbounded sectors.Fan[10] extend above resultsto multivariable monotonenonlinearities,as well as relaxing observer feasibility conditions via multiplier by exploiting the decoupled nature of the multivariable nonlinearity. In this article ,The system which we consider consists of a linear time invariant part and a nonlinear time-varying part. The nonlinear part is a multivariable monotone nonlinear.In this paper, a set of symmetric matrixs are used to generalize the nonlinear time-varying part. These matrixs are also called the increment multiplier matrixs.This article will describe the nonlinear part by means of matrix inequality. The inequality is called the incremental quadratic constraints.The above inequality is parameterized by the incremental multiplier matrix of the nonlinear part.

In this paper, we will make full use of the observer design method which appears in Fan[10] and Açıkmeşe[11]. The systems for our consideration,the observer whose framework is inspired by Arcak[6],Açıkmeşe[11]and Corless.Above observers can be characterized . We also consider the problem of simultaneously computing L and Ln. We usually give specific conditionson the set of incremental multiplier matrices describing the nonlinearities, the problem of simultaneously determining L and L_n translate into lmis.

The organization of this article is as follows.In section 2,we will recall the quadratic stability. In section 3,We define the class of nonlinear systems . And formally state the impersonal of the article.Consequently, the observer structure is presented and sufficient conditions are provided . Section 4,we will offer conclutions.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, we consider the class of nonlinear systems described by the following nonlinear state equations:

$$\begin{aligned} \dot{x} &= Ax + B\gamma_\gamma(t,x,u) + g(w) \\ y &= Cx(t) \end{aligned} \qquad (1)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^q$ is the measurable output, $t \in R$ is a time variable and $w(t) = (t, u(t), y(t))$.

$\gamma_\gamma(\bullet)$ which satisfy the monotone of multivariable simulation[10]:

$$\frac{\partial \gamma_\gamma}{\partial \upsilon} + \left(\frac{\partial \gamma_\gamma}{\partial \upsilon}\right)^{\mathrm{T}} \geq 0 \ \forall v \in R^p \qquad (2)$$

All nonlinear element in the system are put into γ_γ and g . We assume

that $\gamma_\gamma(t,x,u) \in R^p$ is given by

$$\gamma_\gamma(t,x,u) = \varphi(w,q) \text{ which } q = h_q x + d_q \gamma_\gamma \qquad (3)$$

φ is a continuous function,and $q \in R^q$ .The mattrices A, B, C, h_q, d_q is continuous and suitable dimensions. Due to the movement of the device, $x(t) : [t_{o,} +\infty) \in R^n$,which is appropriate for (1) .

Definition 1(Incremental multiplier matrix)

A symmetric matrix M which is an incremental multiplier matrix [12]for γ_{γ} if it meet the incremental quadratic constraint

$$\begin{bmatrix} \delta q \\ \delta\varphi \end{bmatrix}^T M \begin{bmatrix} \delta q \\ \delta\varphi \end{bmatrix} \geq 0 \qquad (4)$$

where $\delta q = q2 - q1$ (5a)

and $\delta\varphi = \varphi(w, q1) - \varphi(w, q2)$ (5b)

for $q2, q1 \in R^q$;

In order to illustrate the concept of δQC ,We give several examples

Example one

Consider any monotone scalar valued function φ of a scalar variable,

which is $\varphi(q) \geq \varphi(\hat{q})$ and $q \geq \hat{q}$

it is equal to $(\varphi(q) - \varphi(\hat{q}))(q - \hat{q}) \geq 0$

for all $q, \hat{q} \in R$.We can find that above inequality can be converted to following inequality which meet δQC.

$$\begin{bmatrix} q - \hat{q} \\ \varphi(q) - \varphi(\hat{q}) \end{bmatrix}^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q - \hat{q} \\ \varphi(q) - \varphi(\hat{q}) \end{bmatrix} \geq 0$$

So,an incremental multiplier matrix for φ is

$$M = k \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

for all $k > 0$,

Example two

Consider the nonlinear function which is a Lipschitz nonlinearity with a Lipschitz constant β.

$$\| f(x1) - f(x2) \| \leq \beta \| x1 - x2 \|$$

we have $(f(x1) - f(x2))^2 \leq \beta^2 (x1 - x2)^2$

that is $$\begin{bmatrix} x1 - x2 \\ f(x1) - f(x2) \end{bmatrix}^T \begin{bmatrix} \beta^2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x1 - x2 \\ f(x1) - f(x2) \end{bmatrix} \geq 0$$

Hence an incremental multiplier matrix for $f(x)$ is

$$M = \kappa \begin{bmatrix} \beta^2 & 0 \\ 0 & -1 \end{bmatrix}$$

for any $\kappa \geq 0$

Hypothesis 1. There is a continuous function ψ_p such that ,for all

w, z

$$\gamma_\gamma = \psi_\gamma(w, z) \quad \text{where} \quad z = h_q x \qquad (6)$$

The above hypothesis produced that

$$\psi_\gamma(w, z) = \varphi(w + z_q \psi_\gamma d) \qquad (7)$$

The above inequality is an implicit function, which can be applied to many occasions. For instance, $\dot{x}_1 = x_1 + 3x_2, \dot{x}_2 = \tan(x_1 + \dot{x}_2)$ .Letting $\gamma_\gamma = \tan(x_1 + \dot{x}_2)$,we can get $\dot{x}_2 = \gamma_\gamma$ .Let $q = h_q x + d_q p$,where $c_q = [1 \quad 0]$ and

$d_q = 1$. At last $\gamma_\gamma = \tan(z + \gamma_\gamma)$;which can be denoted by $\gamma_\gamma = \psi_\gamma(z)$;

III. OBSERVER DESIGN

First of all, we need to construct a common observer structure by (1) the above section:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B\hat{\gamma}_\gamma + g(w) + L(\hat{y} - y) \\ \hat{y} &= C\hat{x} \\ \hat{\gamma}_\gamma &= \psi_\gamma(h_q \hat{x} + K_1(\hat{y} - y)) + K_2(\hat{y} - y) \end{aligned} \qquad (8)$$

Our mission is to design observer gain matrices L, K_1 and K_2 which are appropriate constant matrixs . These matrices guarantees that the state estimate $\hat{x}$ of this observer gradually estimate the system's state in exponential form.The first amendment, $L(\hat{y} - y)$ is linear output error section.The second term, $K_1(\hat{y} - y)$ is nonlinear input part, it has appeared in many articles. At last , $K_2(\hat{y} - y)$ is an additional feature to the observer design.

3.1 The condition of the observer gain for the asymptotic estimation

The following theorem which provides conditions on above observer gain matrices L, K_1 and K_2 which can guarantee the exponential convergence of the observer state error to zero.

Theorem 1. Assume that there exit a decay rate $a > 0$ and Lyapunov matrix $P = P^T > 0$, if the matrix inequality form is :

$$\begin{bmatrix} P(A+LC)+(A+LC)^T P + 2aP & PB \\ B^T P & 0 \end{bmatrix} + \Phi^T M \Phi < 0 \qquad (9)$$

where $$\Phi = \begin{bmatrix} h_q + (K_1 - d_q K_2)C & d_q \\ -K_2 C & I \end{bmatrix} \qquad (10)$$

Then above observer(8) is good defined which state error is exponential convergence :

$$|e(t)| \le k(P)|e(t_0)| \exp^{-a(t-t_o)} \qquad (11)$$

where $e = \hat{x} - x$ and $k(P) = \sqrt{\lambda_{\max}(P)/\lambda_{\min}(p)}$,

$\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ express the largest and smallest eigenvalues of P .

Proof. The $x(t) \in [t_0, t_1)$ is any movement of the device , $\hat{x}(t_0) = \hat{x}_0$ is initial condition of observer .The state $\hat{x}$ of the observer is described by $\dot{\hat{x}} = (A+LC)\hat{x} + B\gamma_\gamma + g(w) - LCx$ .The state error can be described by

$$\dot{e} = \dot{\hat{x}} - \dot{x} = Ae + B(\hat{\gamma}_\gamma - \gamma_\gamma) + LCe = (A+LC)e + B\delta\gamma_\gamma \qquad (12)$$

where $\delta\gamma_\gamma = \hat{\gamma}_\gamma - \gamma_\gamma$.

Since that $\hat{\gamma}_\gamma = \psi_\gamma(h_q\hat{x} + K_1(\hat{y}-y)) + K_2(\hat{y}-y)$ and $\gamma_\gamma = \psi_\gamma(w,z)$,we can get that $\delta\psi_\gamma = \psi_\gamma(h_q\hat{x} + K_1(\hat{y}-y)) - \psi_\gamma(h_q x) = \hat{\gamma}_\gamma - \gamma_\gamma - K_2(\hat{y}-y) = \delta\gamma_\gamma - K_2 Ce$ .The M is incremental multiplier matrix for φ if and only if N (12) is an incremental multiplier matrix for ψ_γ .

$$N := \begin{bmatrix} I & d_q \\ 0 & I \end{bmatrix}^T M \begin{bmatrix} I & d_q \\ 0 & I \end{bmatrix} \qquad (13)$$

We can find the relationship between ψ_γ and φ :

$$\begin{bmatrix} q \\ \varphi \end{bmatrix} = \begin{bmatrix} I & d_q \\ 0 & I \end{bmatrix} \begin{bmatrix} z \\ \psi_\gamma \end{bmatrix} \qquad (14)$$

When $d_q = 0$, ψ_γ is equal to φ .So(4) can be converted into (15)

$$\begin{bmatrix} z_2 - z_1 \\ \hat{\psi}_\gamma - \psi_\gamma \end{bmatrix}^T N \begin{bmatrix} z_2 - z_1 \\ \hat{\psi}_\gamma - \psi_\gamma \end{bmatrix} \geq 0 \qquad (15)$$

The above incremental quadratic stability inequlity with $z_1 = h_q x$ and $z_2 = h_q \hat{x} + K_1(\hat{y} - y) = h_q x + h_q e + K_1 Ce = h_q x + (h_q + K_1 C)e$ .So $\delta z = (h_q + K_1 C)e$.

We construct Lyapunov function $V = e^T pe$.$\dot{V} = \dot{e}Pe + eP\dot{e} = 2e^T P\dot{e}$.Combining

above equation and (12).We can conclude that $\dot{V} \leq 2aV$ .If functions are satisfied with (9) .The state error is exponential convergence.

Example: We propose the following nonlinear system which appeared in [13]under the stucture:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\delta \end{bmatrix}, B = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \gamma_\gamma = x_1^3, g(w) = \begin{bmatrix} 0 \\ \sigma \sin t \end{bmatrix}$$

where $\delta = 0.1, \sigma = 10.5$ and $x = [x_1 \quad x_2]^T$.

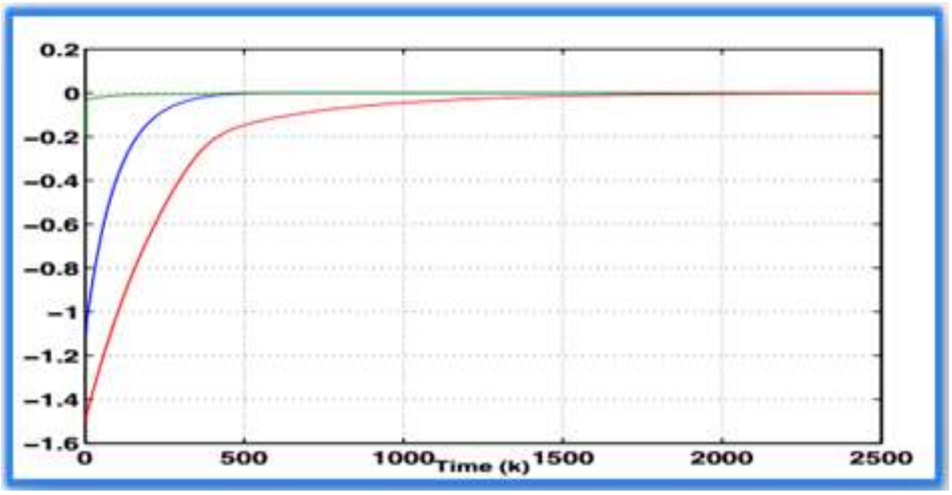

Fig. 1. The estimation error behavior.

$$L = \begin{bmatrix} 191 & 456 & 42 & 335 \\ -365 & 4589 & -397 & -795 \end{bmatrix}, K_1 = I \text{ and } K_2 = 0.$$

The exponential convergence of the state estimates error is explained with simulations in figure 1.

IV. CONCLUTIONS

We developed a method for observers with multivariable nonlinearities satisfying incremental quadratic stability.We formulate linear matrix inequlities which can be used to construct the above observer.It is a good way to determine state error exponential`l convergence.

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